Investigation of co-rotating vortex merger in ground proximity

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The experiments were conducted in a water tunnel facility and were aimed at investigating the merging process of co-rotating wake vortices in ground effect. A corresponding two-dimensional analytical model was also introduced based on experimental results. The two like-sign vortices were generated with equal size and strength above the ground plane. The evolution of vortex pair was attained by using particle image velocimetry at a series of downstream locations. The main finding is that the ground effect promotes the merging process and accelerates the vortices' rotation around one another. Vertical movements, known as vortex rebound, and lateral movements are observed. The ground effect also leads to the emergence of a secondary vortex (SV), which adds to the complexity of the vortex system. Prediction of the modified potential model, which takes vortex decay, viscous drag and the evolution of the SV into account, is compared with the experimental data. The results show that the vortex merging and rebound of a co-rotating vortex pair is successfully predicted and the prediction of vortex orientation and lateral motion are also improved by this model.

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1. Introduction

For an aircraft wing to generate lift, air flows from the lower surface to the upper surface around the wing tips and high-lifting surfaces. Wake vortices generated by lifting surfaces consist of span-wise vorticity sheets that shed rapidly and roll up into distinct vortex structures. The wake vortices persist for a long period of time, possessing a high amount of kinetic energy and thus, pose a potential hazard to following aircraft. To alleviate wake-vortex encounter risk, regulatory separation distances among aircraft weight classes are required [4], which directly limits the potential handling capacity of an airport. Hence, investigations into wake-vortex behaviour have been increasing in the aviation industry in recent years [7]. Wake vortices are strongest and most hazardous on congested airfields during take-off and landing, i.e. in close proximity to the ground [30]. It is held that serious interactions between wake vortices and the ground can significantly affect the evolution of wake vortices, thus making them difficult to predict. Hence, in order to make accurate predictions of wake development on an airfield, better understanding of the dynamics of wake multi-vortices systems as well as the interactions between wake vortices and the ground are required.

Stephan et al. [28] reported that during approach and landing co-rotating vortices were formed and merged into a single vortex on each side of the wing. These vortices led to the formation of a shear layer at the ground, which eventually rolled up a secondary vortex structures and separated from the ground. Fig. 1 shows a schematic drawing of a typical trailing vortex system emanating from an aircraft during take-off or approach. It can be seen that the co-rotating vortex pairs with comparable strength that sheds from the wing tips and the deployed flaps dominate the extended near field of the trailing-edge wake [4,14]. The co-rotating vortices spin around each other and eventually merge over a distance of 5–10 wing spans [19]. It is believed that the manner of the co-rotating vortex merger can determine whether the resulting vortical wake system remains discrete or becomes more diffused [25]. Meanwhile, the co-rotating vortex system induces span-wise separation on the ground, which develops and forms a SV. The formation of the SV will also influence the development of the co-rotating vortices and the successive counter-rotating vortex pair.

There have been many researches into the co-rotating vortex merging process. Meunier and Leweke [20] divided the whole merging process of a pair of equal strength co-rotating vortices into three stages, with Cerretelli and Williamson [5] adding a fourth stage to describe the end of merging. The four stages were named as the first diffusive stage, convective merging stage, second diffusive stage and merged diffusive stage, respectively. In the first diffusive stage, the co-rotating vortex pair undergoes diffusive growth, with the vortices rotating around each other due to mutual induction and the vortex separation remains constant. When the vortices grow large enough, the antisymmetric vorticity induced

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by another vortex gradually diffuses into the outer region leading to the onset of the convective merger, causing the two vortices to move towards each other [17]. A critical ratio between the vortex core and vortex separation at the onset of this stage has been found to be 0.24–0.31 for merging without instability and 0.19 with instability [5,19]. In addition, it has also been found that the decay of the vortex system accelerates during this merger stage [5]. Towards the end of the convective stage, the induced merging velocity from diminished antisymmetric vorticity becomes small and there is the final merging of the vortex pair into one vertical structure by vorticity diffusion, which is termed the second diffusive stage. As the merging process is completed, the wake system of the aircraft ends up with a pair of counter-rotating vortices in the far field, as shown in Fig. 1.

The interaction between the co-rotating vortex pair and the ground effect in Fig. 1 can also affect the merger and the initial status of the counter-rotating vortex system. Extensive numerical and experimental studies have been performed on the evolution of single/counter-rotating vortex/vortices with ground effect. It has been suggested that the ground effect not only induces lateral shifting and vertical rebound [27], but also influences the development and decay of the vortex system [13]. Notably, the interaction between the co-rotating vortex system and the ground has not received much attention in the technical literature and hence, the salience of the present study. Prior to discussing the ground effect on co-rotating vortex merger, a brief review of the literature on the general effect of ground proximity and approaches to modelling wake flow is presented.

First of all, in order to distinguish the different levels of ground proximity, Robins et al. [24] divided the near-ground counter-rotating vortex flowfield into three regimes, namely, out-of-ground effect (OGE), near-ground effect (NGE) and in-ground effect (IGE), depending on the height of the vortices. For the case of counter-rotating vortices at heights above approximately 1.5 times the initial vortex separation, no interaction between the vortices and the ground is observed and consequently, the counter-rotating vortex pair sinks at a constant rate due to their mutual interaction. The vortices are thus said to be OGE. NGE is a transition region between the OGE and IGE with the wake heights defined as 1.5–0.5 times the vortex separation. Within the NGE, the dynamics of counter-rotating vortices are still dominated by mutual induction, while the viscous effect between the vortices and the ground is weak and therefore, considered to be negligible based on the suggestions of previous studies [13,24]. When the vortices sink below approximately half-span, they become subject to the in-ground effect, IGE. In this region, the interaction between the vortices and the ground leads to an adverse pressure gradient that makes the spanwise boundary separate, which is followed by the formation of a SV [22].

There have been a number of research projects concerning the behaviours of the counter-rotating vortex pair close to the ground or the wall. Jones [15] modelled the motion of a pair of near-wall counter-rotating vortices produced by vortex generators and showed that by adding an image vortex below the wall the lateral movement could be predicted. Later, Harvey and Perry [9] suggested that the PV leads to the development of a transverse boundary layer at the wall, while a strong adverse pressure gradient is created within this boundary layer, which makes it separate and gives rise to a SV. The vorticity of a SV is opposite to the PV and pushes the latter up, which is termed vortex rebound. This explanation has been confirmed by later water tank flow visualization [26], numerical simulation [3] and airport field measurement [13]. In addition to the SV, the creation of tertiary and quaternary vortices has been observed, which obliges the PV to rebound several times [21]. Puel and Victor [22] simulated two-dimensional and three-dimensional wake vortices near the ground using incompressible Navier–Stokes equations. It was confirmed that the SV pushes up the PV and leads to vortex rebound in a looping of trajectories.

Kliment and Rokhsaz [16] investigated the motion of a co-rotating vortex pair in ground effect and confirmed the lateral motion and vortex rebound, but elicited little information regarding the merger mechanism. In their research, a potential model that introduced a pair of image vortices below the ground was used to
simulate the motions of vortices, which could not predict the vortex rebound. Furthermore, no merger mechanism was considered in their co-rotating vortex model and the predicted trajectories of vortex evolution became unrealistic, thus leaving substantial room for the potential model to be improved. In the present paper, the co-rotating vortex merger is investigated in relation to the influence of ground proximity. Both experimental and analytical works are demonstrated. Flow features were observed in experiments, including the SV and vortex decay, which were later quantified and carefully added into the potential model. These offer significantly more correct predictions regarding the evolution of IGE extended near wake vortices in a relatively less expensive numerical approach. After further work, the authors’ intention is to add the vortex merging physical features to a wake-vortex advisory system (WVAS) [10–13], thus allowing for its application to the extended near field of the wake system, which could provide a better estimation of the initial status of the long-lasting counter-rotating wake vortices.

2. Experimental methods

The experiments were conducted in a free-surface closed-loop water tunnel at the Beijing University of Aeronautics and Astronautics. The water tunnel is capable of flow speeds in the range of 0 to 0.5 m s\(^{-1}\) and has a test section size of 525 mm (H) × 400 mm (W) × 4000 mm (L). During the test, the water tunnel was operated at a constant freestream velocity \(U_\infty = 0.105\) m s\(^{-1}\). The turbulence level for this operating condition was estimated to be less than 1\% of the freestream velocity. The water temperature was 23 ± 0.3°C with a kinematic viscosity of \(\nu = 0.9347 \times 10^{-6}\) m\(^2\) s\(^{-1}\). The co-rotating vortex pair was generated by using two fully submersed NACA0012 rectangular wings horizontally mounted at opposite angles of attack, as shown in Fig. 2. The wing models were made out of aluminium alloy with a chord length \(c = 100\) mm and an aspect ratio of 1.5. The wing tips were rounded in a semi-circular profile.

An extended splitter plate, namely, a ground plane, was placed 300 mm downstream from the trailing edge of the vortex generators so as to apply the ground effect. The ground plane had a round leading-edge and elliptical profile with the physical dimensions being 2400 mm long, 398 mm wide and 8 mm thick. Clearance between the ground plane and each side of test section vertical wall was maintained at 1 mm (0.01c). The upper surface of the ground effect plate was covered with matt black laser-absorbent paint to reduce the reflection created from the laser sheet during particle image velocimetry measurement. The coordinate system of the current study can be found in Fig. 2, where \(x, y, z\) represent the coordinates of the streamwise, lateral and vertical directions, respectively. The coordinate origin was chosen as the midpoint between the vortex centres (vorticity maxima) of two vortices at \(x/c = 0\) (at the streamwise location parallel to the leading edge of the ground plane).

A co-rotating vortex pair is characterized by the following parameters, and shown diagrammatically in Fig. 3. The primary vortex near the ground plane is denoted as \(PV_A\) and \(PV_B\) the remote one. The vortex separation \(b\) was defined as the separation distance between the vortex centre of the two \(PV\)'s, while \(b_0\) was the initial vortex separation at \(x/c = 0\). In the current research vortex core radius \(a\) is defined as the distance from the vortex centre to maximum tangential velocity and the circulation \(\Gamma\) is computed from a contour line integral of the velocity around each vortex. Varying the angles of attack of the rectangular wings allowed for manipulation of the value of \(a\) and \(\Gamma\). The initial vortex core radius \(b_0\) was approximately 0.066 m, with the initial circulation being \(\Gamma_0 = 1.32 \times 10^{-3}\) m\(^2\) s\(^{-1}\) and the vortex core Reynolds number was \(Re\_\Gamma = 1.42 \times 10^3\). The rotation \(\theta\) is the angle that the vortices rotate around one another, also known as the vortex pair orientation. In order to minimize the interference between the rectangular wings and to ensure that both vortices were generated with similar size and strength, the above parameters were carefully selected. In the IGE cases, the ground plane was placed at the vertical location of \(z/b_0 = -1\) and \(h\) was the height of vortex (vertical distance between the vortex centres and the ground plane). At \(x/c = 0\), both vortices were chosen to have the same initial height of \(h_0/b_0 = 1.0\), based on the comparable height to separation ratio of a transport aircraft wake parameter during take-off [4]. The values of the initial parameters of the co-rotating vortex pair are shown in Table 1. Four cases of initial arrangements with vortex core radius to separation ratios of \(a_0/b_0 = 0.124\) and 0.145 were tested in both the OGE and IGE conditions.

PIV measurements were taken at various cross-flow locations (from the leading-edge of ground plane \(x/c = 0\) to downstream locations with an interval of one chord length, \(\Delta x/c = 1\) using a Dantec 2-D PIV system consisting of dual 200 mJ Nd:YAG lasers and a 12 bit CCD camera with a resolution of 2048 × 2048 pixels.

<table>
<thead>
<tr>
<th>(a_0/b_0)</th>
<th>(Re_\Gamma)</th>
<th>(b_0) (m)</th>
<th>(a_0) (m)</th>
<th>(h_0) (m)</th>
<th>Configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.124</td>
<td>1.42 × 10^3</td>
<td>0.0532</td>
<td>0.0066</td>
<td>N/A</td>
<td>OGE</td>
</tr>
<tr>
<td>0.145</td>
<td>1.42 × 10^3</td>
<td>0.0460</td>
<td>0.0067</td>
<td>N/A</td>
<td>OGE</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0457</td>
<td>0.0067</td>
<td>1.0</td>
<td>IGE</td>
</tr>
</tbody>
</table>

Fig. 2. Schematic layout of the water tunnel facility.

Fig. 3. The definition of the vortex parameters in the current experiments.
from the downstream location, as shown in Fig. 2. A synchronizer unit was utilized to link the camera and laser so as to enable accurate capture for the two frame cross-correlation analysis. The flow was seeded with commercially available hollow glass particles with a mean diameter of 1–4 μm and the system was operated at a sampling frequency of 1 Hz in the cross-correlation mode. The PIV images were analyzed using the Dantec Dynamic Studio software using an adaptive correlation on an interrogation window size of 32 x 32 pixels and the effective grid size varied from 0.5 to 0.6 mm in these measurements. The time-averaged data was derived from 300 pairs of images and the estimated uncertainty for the velocity measurements was within 5%.

Flow visualization was carried out by illuminating the fluorescent dye with a 10 W continuous diode pump solid state laser which emitted a 532 nm wavelength laser sheet. Yellow and green dyes were used to distinguish the PV and SV, respectively. To capture the structure of the unsteady SV area, the green dyes were blended with aloe vera gel and injected into the flow from the ground plane with carefully controlled exit velocity. To ensure that the dye did not affect the formation of the SV the dye exit was planted on the upper surface of the ground plane at the front left side of the PV pair. The dye was transported along the ground plane by the induction of the PVs and eventually highlighted the shear layer and the SV structures.

3. Co-rotating vortex in ground effect

The cross-flow PIV measurements of the co-rotating vortex pair merger with and without ground effect are shown in Fig. 4. The evolutions of the time-averaged normalized vorticity contours are illustrated for both the OGE and IGE cases with \( \alpha_0/\theta_0 = 0.124 \) at \( x/c = 0, 5, 10 \) and 15. The vorticity contour level was selected in a range between -2 and 12 with an interval of 0.5, where the positive vorticity rotated counter-clockwise. Note that, in order to improve the identification of the vorticity contours and highlight the vortical flow features, vorticity between -0.5 and 1 was not shown in the contour. The upper row (OGE case) showing the merger of the co-rotating vortex without ground effect is the baseline case of the current study, while the lower row (IGE case) demonstrates the merger of the co-rotating vortex pair in ground effect.

In Fig. 4 the OGE co-rotating pair rotated around each other with vortex separation remained constant. The two vortices in OGE case remain separated from each other during rotation and the centre of the vortex pair stays around the original point. In contrast, in the IGE case the co-rotating vortex pair was found to have significantly different merger behaviour from the lower row of Fig. 4. The vortex pair shifted laterally and upward compared to the OGE case, indicated by the deviation of the centre of vortex pair from the original point, which is typical for IGE vortex evolution and has been explained as owing to the results of potential-flow effects and the SV in several previous pieces of research (e.g. [16,22]). Moreover, accelerated rotation and decreased vortex separation were observed in the IGE case. At \( x/c = 10 \) the vortex separation reduced down to 0.8\( \theta_0 \) in the IGE case and the size of both vortices increased significantly. By comparison, in the OGE case both the separation and dimensions of the vortex pair remained almost unchanged. At \( x/c = 15 \), the merger of the IGE vortex pair occurred. The ground effect also led to the emergence of a SV, which is believed to be the main reason for vortex rebound [5,21,22]. It is noticed that, at \( x/c = 5 \), as the vortices rotated around one another and approached the ground, the induced flow near the ground plane surface experienced an adverse pressure gradient, which was strong enough to create flow separation and later gave rise to the second vortex with opposite-sign vorticity at \( x/c = 10 \).

Fig. 5 demonstrates the ground effect on the evolution of a co-rotating vortex pair quantitatively through providing the vortex separation, angle of orientation as well as the lateral and vertical positions of the vortices. In Fig. 5a, the vortex separations of the IGE cases are smaller than those in the OGE cases at the same streamwise locations for both \( \alpha_0/\theta_0 = 0.124 \) and 0.145, from which the vortex merging is hereby considered to be promoted due to the ground effect. For the OGE case with \( \alpha_0/\theta_0 = 0.124 \), the vortex separation being maintained above its initial value throughout all tested streamwise locations, is believed to be due to disturbances of the wake flow of the vortex generators. In previous experimental research [19,20], it was also observed that the interactions between the vortex pair and wake from vortex generators could result in 10% to 20% \( \theta_0 \) departure of the two vortices, which is similar to the results here. Recall Fig. 4, at \( x/c = 10 \) and 15 the OGE vortex separation remained unchanged and the deformation
of the vortices was less obvious, hence no merger occurred. With \( a_0/b_0 = 0.145 \) rapid decreasing of the vortex separation can be observed at \( x/c \geq 9 \) in the OGE case, which indicates the commencement of the convective merging stage. However, in the IGE case rapid decrease of the vortex separation was observed to be at about \( x/c = 9 \) and 7, with \( a_0/b_0 = 0.124 \) and \( a_0/b_0 = 0.145 \), respectively, which were both promoted compared to the OGE cases.

According to previous work, it is known that the mutual-induced rotation velocity of a vortex pair increases with decreased vortex separation [5,19]. In Fig. 5b, the orientation of the co-rotating vortex pair continuously increases along the downstream locations. It can be seen that with ground effect the rotation of the vortex pair accelerates significantly for both \( a_0/b_0 = 0.124 \) and 0.145, especially when \( a_0/b_0 = 0.124 \), whilst at \( x/c = 15 \) the orientation is almost doubled by the effect of ground proximity. Furthermore, the ground effect also affects the vortex trajectories by causing vertical and lateral movements, as shown in Fig. 5c and Fig. 5d, respectively.

The ground effect has a significant difference on the IGE co-rotating vortex behaviour, so it is necessary to investigate and model the vortex merging and evolution with ground effect. The potential method that treats the vortex as identical point vortices, is a quick and robust numerical approach and has been applied in previous researches on the WWAS (e.g. [13,23,24]). Kliment and Rokhsaz [16] predicted the motion of a co-rotating vortex by the potential method introduced by Hassan [1], in which the vortex positions in the \( y-z \) plane were given by the solution of a set of simultaneous differential equation (1) and the system was assumed to be advected downstream by the freestream. The following Hamiltonian equations describe the motion of the point vortices in such a two-dimensional flow field:

\[
H = - \sum_{j=1}^{N} \Gamma_i \Gamma_j \ln \sqrt{(y_j - y_i)^2 + (z_j - z_i)^2} \\
\Gamma_i \dot{y}_i = \frac{\partial H}{\partial z_i} \\
\Gamma_i \dot{z}_i = - \frac{\partial H}{\partial y_i}
\]

where, \( N \) is the total number of vortices, whilst \( \dot{y}_i \) and \( \dot{z}_i \) give the lateral and vertical velocity, respectively. Kliment and Rokhsaz [16] introduced the images of the PV pair and hence, the total number of vortices is \( N = 2 \) for the OGE case and \( N = 4 \) for the IGE one. Fig. 6 compares the numerical predictions using equation (1) and the experimental results for the current study for both the OGE and IGE cases with \( a_0/b_0 = 0.124 \). The predictions of the OGE case are in good agreement with the experimental results such that the vortex separation was expected to remain unchanged with the absence of real-life disturbances (Fig. 6a). Regarding the current experiments, the vortex departure is believed to be a result of the interaction of the wake flow. When the co-rotating vortex pair was in the IGE case, the predicting method using equation (1) became less compatible. From Fig. 6a, it emerges that the vortex separation decreases all the time and the merger is promoted by the ground effect in the experiments, while the predicted vortex separation starts to increase at \( x/c \geq 8 \), where the two vortices depart from each other instead of merging. Moreover, this method underestimates the vortex angle of orientation (Fig. 6b) and overestimates the lateral movements of the vortex pair (Fig. 6d). It is most notable that, regarding the ground effect the vortex rebound cannot be predicted by such a method, as shown in Fig. 6c. The mean ver-
Fig. 6. Comparison between the experimental results and the predicted results of a previous potential model [16] in the OGE and IGE cases for $a_0/b_0 = 0.124$.

tical position between the IGEPVs is predicted as being $z/b_0 = 0$ along all streamwise locations.

Consequently, a more compatible numerical model is required, which can provide a better prediction of co-rotating vortex pair evolution and merger in ground proximity.

4. Model concept

4.1. Modelling of the secondary vortex

The current experimental results have revealed that the ground effect can affect the merging behaviour and trajectories of the co-rotating vortex pair. However, the available potential model [16] is less compatible and fails to predict the IGE flow phenomenon, for instance regarding the promoted vortex merger and the vortex rebound (Fig. 5). A typical IGE co-rotating vortex pair flow field can be found in Fig. 4 where a SV is observed. It is known that the SV is a major factor of the vortex rebound and affects the evolution of IGE vortices [21,22], hence it is necessary to consider it in the current predicting model.

Due to the fact that the SV is unsteady and wanders around the mean position, hereby the time-averaged vorticity contours of the SV are less identical at $x/c = 10$ and 15 in the PIV measurements in Fig. 4. Consequently, laser dye flow visualizations were given to illustrate the generation and development processes of the SV as well as its interactions with the primary vortices, as shown in Fig. 7, where the yellow represents the PVs and the green is the SV. In Fig. 7a, the cross-flow separation bubble (indicated by the bump of the green dyes on the ground plane) was formed beneath the PV pair, because the induction of the primary vortex led to a cross-flow adverse pressure gradient. As the flow separation further developed, the negative vorticity sheet detached from the ground plane and formed a SV (Fig. 7b). At a further downstream location the SV was dispersed from the ground plane and taken into the upper flow field by the PVs, as shown in Fig. 7c. It is noticed that the formation of the SV is slightly delayed when compared to the PIV results, which is considered to be the consequence of the additional viscous effects imposed by the aloe vera gel.

The effect of such a detached SV can be modelled by adding point vortices and its image to the system [24]. So, in the current study the number $N$ in equation (1) is set to 2 in the OGE case and $N = 6$ in the IGE ones. The first SV is initially introduced beneath the centre of the vortex pair with an initial circulation of zero (Fig. 8a). The circulation of the SV is a function of its orientation $\theta_{1}(\text{Fig. 8b})$, which grows with increasing orientation and reaches maximum strength at $\theta_{1} = 45$ degree followed by a linear drop in which the circulation becomes zero at $\theta_{1} = 180$ degree. Once the first SV has diminished the second SV is added (Fig. 8c) to the system. According to previous researches on counter-rotating vortices in ground proximity, the circulation of the SV is often considered as a ratio of the initial circulation of the PV [10,13,24]. Table 2 shows the typical ratios between maximum SV circulation and PV initial circulation in these researches. It is noted that such a ratio was determined by observation and differs amongst cases. In the current study, the circulation ratio can be determined by referring to the experimental results.

Fig. 9 gives the variation of the ratio between the circulation of the SV and PV in the IGE cases for $a_0/b_0 = 0.124$ and 0.145. Note that, for the experimental results only the normalized vorticity of below $-0.5$ is integrated to calculate the circulation of the SV. As demonstrated in Fig. 9, the circulation of the SV increases initially and starts to decrease once the ratio reaches its peak value.
Fig. 7. The development of the co-rotating vortex pair and the SV shown by dye visualizations of the \textit{IGE} case with $a_0/b_0 = 0.145$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Table 2
The ratio between secondary vortex maximum circulation and primary vortex initial circulation in the previous research on \textit{IGE} counter-rotating vortex pair.

<table>
<thead>
<tr>
<th>Former researchs</th>
<th>$\Gamma_{s,\text{max}}/\Gamma_0$</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>R.E. Robins et al. [24]</td>
<td>0.4</td>
<td>Symmetric</td>
</tr>
<tr>
<td>F. Holzäpfel [10]</td>
<td>0.4</td>
<td>Symmetric</td>
</tr>
<tr>
<td>F. Holzäpfel [13]</td>
<td>0.2(luff)-0.4(lee)</td>
<td>Asymmetric (affected by cross wind)</td>
</tr>
</tbody>
</table>

at $\Gamma_{s,\text{max}}/\Gamma_0 \approx 0.6$. It is found that the circulation ratio value in the current study is greater than previous values presented in Table 2. The main reason for such a difference is believed to be that the co-rotating vortex pair induces greater transverse velocity on the ground beneath the PVs as compared with the counter-rotating vortex pair case. Moreover, it is also found that for the vortices system with a smaller $b_0$ (or a smaller $h_0$), an earlier formation of the SV follows with a quick decay occurring, which is probably as a result of a faster rotating SV around the PV pair. The solid and dash lines are the results of the predicted circulation ratios between the SV and PV which are in good agreement with the experimental results.

Fig. 8. Schematic of the evolution of the secondary vortex in prediction.

Fig. 9. The ratio of the circulation of the secondary and primary vortices for the \textit{IGE} cases.

4.2. The effect of viscous drag and vortex decay

Besides the SV, there are several other physical ingredients which might affect the evolution of the co-rotating vortex in ground effect, such as vortex decay, viscous drag, temperature
stratification, cross-wind, etc. [8,13]. Due to the fact that there are certain difficulties in demonstrating the temperature stratification and cross-wind with the current experimental setup and numerical model, only vortex decay and viscous drag are considered here.

When the vortices move in the flow field, they are subjected to viscous drag. An empirical estimate of their magnitude was made by assuming that the viscous drag acting on the circular vortex core would be the same as those acting on a similar solid cylinder with the same radius [8]. With this assumption, the viscous drag per unit length on the vortex core is \( -\rho V^2/2 \cdot C_D \cdot 2a \), where \( V \) is the vortex crossflow \((y-z\) plane\) travelling velocity and \( C_D \) is the drag coefficient, which can be determined by the drag of the cylinder. The Reynolds number, based on the vortex travelling velocity \( Re_V = V \cdot 2a/\nu \), is less than \( O(10^2) \) and so, \( C_D \) is set to a typical value of 2.0 [29]. The changing rate of unit momentum \( I = \rho \pi a^2 V \) is

\[
\frac{dI}{dt} = \frac{\rho \pi a^2}{\pi a} \frac{dV}{dt} = \frac{-\rho V^2}{2} C_D 2a
\]

And the changing rate of velocity can be determined by:

\[
dV = -\frac{V^2}{\pi a} C_D \frac{dt}{dt}
\]

where, \( dV \) can be added to equation (3) to calculate \( \dot{y_i} \) and \( \dot{\zeta_i} \).

To apply the viscous drag in the current model the vortex radius in equation (2) needs to be determined. The vortex is thus considered as a Lamb–Oseen vortex [2] and the radius growth rate can be represented in the form \( a^2 = a_0^2 + C_{1-0} \cdot \nu \cdot t \), where \( C_{1-0} \) is the Lamb–Oseen vortex growth rate parameter [5] (a theoretical value 2.24 is adopted here) and \( \nu \) is the effective kinematic viscosity which defines the diffusion rate of the vortex. Fig. 10 illustrates the growth of the vortex radius for the IGE cases with effective kinematic viscosity. Initially, the vortex core gradually grows with a small effective viscosity, but when the merging process begins this becomes greater. Specifically, the effective viscosity is \( \nu \approx 1.1 - 1.2 \times 10^{-8} \) prior to merging, while during merging it is about four times greater, which leads to a rapid increase of the vortex radius, as shown in Fig. 10. Since the initial core radius and the effective viscosity are assigned based on the experimental results, the evolution of the core radius can be determined. The viscous effects play an important role in instability, turbulence and advection, etc. However, the current model is a quick prediction algorithm based on a potential method and so only the viscous drag of vortex motion is considered, which is a compromise between the prediction accuracy and the complexity of the model.

Decay of the \( PV \) is observed in the experimental results (Fig. 4), which means that the circulation term \( I^* \) in equation (1) is decreasing with time. During vortex merging, the circulations of the

\[
\frac{dI^*}{dt} = -0.82 \frac{q I^*}{b}
\]

where, \( q \) is the root-mean-square fluctuation velocity and the constant \(-0.82 \) is an empirical value given by Donaldson and Bililan [6] as a “best guess”. Taking the IGE case of \( a_0/b_0 = 0.124 \) as an example (Fig. 11), the influence of flow turbulence in terms of \( q \) on vortex decay is applied by adding equation (4) to determine the circulation of the \( PV \) at streamwise location \( x/c > 9 \). It is obvious that a greater \( q \) leads to faster decay of the vortices as shown in Fig. 11, which coincides with Marles and Gursul’s observation [18] that higher turbulence level can lead to a more rapid decay of the co-rotating vortices. Fig. 12 shows the normalized circulation (averaged over two vortices) of the experimental and the numerically predicted circulation in the IGE cases based on equation (4) where the average value of \( q \) is 0.031 and 0.028 for \( a_0/b_0 = 0.124 \) and 0.145, respectively. The vortex decay is triggered after the onset of merging, which is defined as when the ratio between the vortex
radius and vortex separation exceeds a typical value of 0.24, based on the experimental results. The vortex decay predictions match well with the experiments despite being slightly offset at the onset of quick circulation decrease. There is another method for modelling the decay of the primary vortex by taking the dissipating rate into consideration \[13,24\], which could be an alternative for predicting the decay of the IGE co-rotating vortex pair. However, sufficient evidence from experiments that validate this method is yet to be acquired.

4.3. Prediction results and discussion

For the current work, a \(SV\), viscous drag and vortex decay were incorporated into a model of the IGE co-rotating vortices and the parameters were defined quantitatively based on the current experimental results. When the ratio between the vortex radius and vortex separation exceeds a typical value of 0.24, the \(PVs\) are forced to move towards each other with an empirical laminar merging velocity of \(0.092R_0/b_0\) \[5\], which is currently used to simulate the merging behaviour, as the \(Re\) is below the critical value of 2000 \[5,19\].

Figs. 13–16 compare the vortex separation, angle of orientation, vertical position and lateral position predicted by the present and previous models \[16\] and the experimental results. Fig. 13 shows that the current model can provide a better prediction of the vortex separation. In the IGE case with \(a_0/b_0 = 0.124\) (Fig. 13a), both the experimental and current model’s predicted results suggest a continuous decrease of the vortex separation, in spite of the fact that the previous model reported an increase of the separation at \(x/c > 7\), in which the previous model failed to predict the merger. In Fig. 13b, the results of the IGE case with \(a_0/b_0 = 0.145\) also demonstrate an increase in predicted vortex separation after \(x/c = 6\) for the previous model, while no increase is observed in the experimental and the current model’s prediction. At the location slightly prior to \(x/c = 7\), a rapid decrease of the vortex separation occurred, which coincides well with the experimental results.

In Fig. 13a, the merger at \(6 < x/c < 13\) is overestimated by the current model and by contrast, a vortex separation underestimation is found at \(x/c \geq 13\), with similar results also being found for \(a_0/b_0 = 0.145\), as shown in Fig. 13b. It is believed that such deviation between the current model and experimental results is caused by the way in which the competition between vortex rebound and the lateral motion of the \(PVs\), which will be discussed in a later context.

Fig. 14 shows a comparison between the predicted vortex orientation and the experimental results. The previous model seems to have underestimated the angle of orientation throughout the test, whereas by using the current model there is obvious improvement. This is because adding a \(SV\) to the model not only decreases the vortex separation between the \(PV\) pair, but also adds extra induction to the \(PVs\), which gives rise to accelerated rotation. Yet in case of \(a_0/b_0 = 0.145\) the angle of orientation is overestimated at the end of the merging and its increasing rate is also higher than the experimental results. Recall Fig. 4, during the latter part of the merging process (\(x/c \geq 10\)) it was found that both vortex cores are enlarged and elliptical deformation begins. When \(x/c = 15\), the outer vortical region of the \(PVs\) start to join together and the vortex cores become less identical. In such a situation the
potential method of treating the vortex as isolated point vortices is no longer appropriate and hence, further model development taking into account the vortex interaction and deformation could be required.

In Fig. 15, it is found that the current model improves the prediction of the PV pair vertical movements with vortex rebound. From Fig. 5, it can be seen that the rebound of the centre of the vortex core is about 0.3b₀ for a₀/b₀ = 0.124 and 0.45b₀ for a₀/b₀ = 0.145 at x/c = 16 in the experiments. The current model’s prediction shows good coincidence with the experimental results, while the previous model failed to report any rebound.

It is rather disappointing that both models overestimate the lateral shifts of the PV pair, as shown in Fig. 16. The two models are both based on the potential method that adds the image vortices to the system to satisfy the boundary condition of zero vertical velocity at the ground and yet, the non-slip condition could not be satisfied. In Fig. 4, there is attached negative vorticity distributed as an elongated vorticity sheet beneath the PVs along the ground plane, which was also reported by Stephan et al. [28]. In the current potential method the feature of the attached vorticity sheet cannot be modelled properly, which is believed as the main reason of the overestimation of the lateral motion.

Recall Fig. 13a, a difference in vortex separation between numerical prediction and experiments was found at 6 ≤ x/c < 13 for IGE case with a₀/b₀ = 0.124. A schematic sketch of the lateral and vertical motions of the PVs is illustrated in Fig. 17. The ground-induced vertical velocity components of the two PVs are denoted as VzA and VzB, respectively. It is obvious that VzA is greater than VzB since PV_A is closer to the ground and therefore, receives a stronger rebound. Hence, these vertical motions cause the PVs to approach each other with a velocity difference ΔVz = (VzA − VzB) • sin θ. Note that this difference remains positive at 0 ≤ θ < π, which pushes the PV pair together and thus the merger is promoted. However, the lateral motion of the PVs plays a different role in the merger. The ground-induced lateral velocity components of the PV pair are denoted as VyA and VyB, as shown in Fig. 17. The lateral velocity components result in variation of the relative distance between the PV pair with a velocity difference of ΔVy = (VyA − VyB) • cos θ, which is positive (promoting merger) for 0 ≤ θ < π/2 and negative (delaying merger) for π/2 ≤ θ < π.

It can be concluded that the ground effect causes the induced lateral and vertical movements of the co-rotating vortex pair, which varies the merger process and also explains the predicted vortex separation offset at 6 ≤ x/c < 13. Fig. 18 illustrates both vortex separation and normalized angle of orientation in the IGE case of a₀/b₀ = 0.124. At x/c ≤ 5 the angle of orientation θ in experimental results is less than π/2 and vortex separation decreases due to the combined effect of ground-induced lateral and vertical motion. Once θ ≥ π/2, the lateral motion begins to drag the vortex pair apart, hence under the competition between the lateral and vertical motion, the vortex separation no longer decreases and almost remains constant at around 0.85b₀, until the merging begins at x/c > 10. The predicted results show similar trend, however, slightly underestimated at 6 ≤ x/c < 13 and a “slowdown stage” of vortex separation drop is observed in the range of π/2 ≤ θ < π. Moreover, the predicted results report the vortex orientation exceeds π/2 at about x/c = 7.5, while the position is approximately x/c = 5 for the experimental results. Hence the predicted vortex...
A modified potential model has been proposed by adding a SV, viscous drag and vortex decay to the vortex system based on experimentally verified parameters, which has made significant improvements in predicting the vortex trajectories and the co-rotating vortex merger in ground effect. The involvement of a SV is believed to be the main reason for such improvement.

By adding the SV and its image to the model the vortex rebound can be successfully predicted. It has been found that vortex rebound acts differently between PVs according to their relative distance to the ground, which leads to a difference in the vertical velocity components that decreases of the vortex separation and promotes the merger. The ground effect also induces lateral motion of the PVs, which leads to a decrease in vortex separation for $0 \leq \theta < \pi/2$ and an increase for $\pi/2 < \theta < \pi$. Hence, competition between the vortex lateral and vertical motion occurs at $\pi/2 \leq \theta < \pi$ and the decrease in vortex separation experiences a “slowdown stage”. Furthermore, the ground effect not only generates a detached SV, but also leaves a vortical layer on the ground. By incorporating attach-to-ground vorticity into the model, it is contended here that IGE wake vortical system prediction will be improved.

Conflict of interest statement

There is no conflict of interest.

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References